Finite Element Analysis of Elastic Settlement of Spreadfootings Founded in Soil

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Strength vs. Serviceability (1/3)

• Foundations should be proportioned to withstand all anticipated loads safely including the permanent loads of the structure and transient loads.

• Most design codes specify the types of loads and load combinations to be considered in foundation design, e.g., AASHTO.

• These load combinations can be used to identify the “limit” states for the foundation types being considered. A limit state is reached when the structure no longer fulfills its performance requirements.
Strength vs. Serviceability (2/3)

- **An ultimate limit state (ULS)** corresponds to the maximum load-carrying capacity (either structural or geotechnical failure) of the foundation. The ultimate state is also called the **strength limit state**.
  - bearing capacity of soil exceeded,
  - excessive loss of contact, i.e., eccentricity,
  - sliding at the base of footing,
  - loss of overall stability, i.e., global stability, or
  - exceedance of structural capacity – 1997 UBC or ACI 318

- **A serviceability limit state (SLS)** corresponds to loss of serviceability, and occurs before collapse.
  - excessive differential or total foundation settlements,
  - excessive lateral displacements, or
  - structural deterioration of the foundation.
Strength vs. Serviceability (3/3)

• All relevant limit states must be considered in foundation design to ensure an adequate degree of safety and serviceability. All foundation design in practice is geared towards addressing the ULS and the SLS.

• Existing design methodology includes:
  • the Allowable Stress Design (ASD)
  • the Ultimate Strength Design (USD)
  • the Load and Resistance Factor Design (LRFD)

• Focus is made on ASD for geotechnical design.
The geotechnical design of a spread footing is a two-part process.

1. **Estimate the allowable soil bearing capacity** to ensure stability of the foundation and determine if the proposed structural loads can be supported on a reasonably sized foundation.

2. **Predict an amount of settlement** due to the actual structural loads and the time of occurrence estimated. Experience has shown that settlement is usually the controlling factor in design.

   - This is *not surprising* since structural considerations usually limit tolerable settlements to values that can be achieved only on competent soils not prone to a bearing capacity failure.
Geotechnical design concept and procedure (1/2)

- The allowable bearing capacity of a spread footing is the lesser of
  - The applied stress that results in a shear failure divided by a suitable factor of safety (FS); this is a criterion based on ULS; or
  - The applied stress that results in a specified amount of settlement; this is a criterion based on SLS.
- **Factor of Safety (FS)** = Mean value of Resistance (Material Strength)/Design Load (the maximum load the footing should ever withstand in service); a typical value of FS ranges from 2.5 to 3
- The concept of decreasing allowable bearing capacity with increasing footing width for the settlement controlled cases is an important concept to understand.
- As the footing width increases, the stress increase “felt” by the soil may decrease but the effect of the applied stress will extend more deeply below the footing base. Settlements may increase as the width increases depending on the type of soils within the influence depth.
In such cases, the only way to limit the settlements to a certain desired value is by reducing the applied stress; the allowable bearing capacity is the value of the applied stress at the footing base that will result in a given settlement. The more stringent the settlement criterion the less the stress that can be applied to the footing which in turn means that the allowable bearing capacity is correspondingly reduced. In allowable bearing capacity estimation, a total safety factor of 2.5~3.0 is mostly used.

[Diagram showing allowable bearing capacity vs. settlement]

Allowable bearing capacity line based on ultimate limit state consideration (i.e., no consideration of settlement), $q_{all} = q_{ult}/FS$

Contours of allowable bearing capacity for a given settlement

Effective Footing Width, $b$ (m)

Settlement values

ZONE A: Shear Controls

ZONE B: Settlement Controls

(source: Geotechnical Engineering: Shallow Foundations by Zhou, Y., FHWA NH 06-089)
Structural design concept and procedure

- Foundation design procedures typically provide soil bearing pressures on an allowable stress design (ASD) basis while seismic forces in the 1997 UBC, and in most concrete design under ACI 318, are on an ultimate strength design (USD) basis.

- The designer makes a transition from the ASD procedure to determine a size of the footing to the USD procedures to design the footing.
Example of a combined footing

They are used primarily when the column spacing is non-uniform (Bowles, 1996) or when isolated spreadfootings become so closely spaced that a combination footing is simpler to form and construct.

“Spill-through”
There are various theories to predict (1) shear failure and (2) load-deformation behavior of soil. Why: semi-empirical nature and uncertainty

- Elastic settlement analysis methods:
  - Newmark, Griffiths, Janbu et al., Mayne and Poulos
  - Schmertmann, Meyerhof, Burland and Burbidge
At the present time, various methods are available to calculate the elastic settlement. They are, in general, in two categories; (1) methods based on observed settlement of prototypes that are correlated with in situ tests such as the Standard Penetration Test (SPT), the Cone Penetration Test (CPT), Pressuremeter Test (PMT), and the flat dilatometer test; and Schemertmann’s influence line method (2) methods based on the theory of elasticity such as the Stress Influence Method (Newmark’s solution of Boussinesq Eqn.) and the Strain (displacement) Influence Method

Despite all the extensive library of methods, uncertainties always exist in predicting settlements of the shallow foundation in soil due to highly erratic density and compressibility variation. If soil were elastic, homogeneous, and isotropic, there would be no difficulty in the settlement prediction using the theory of elasticity. In reality, not only are actual soils nonhomogeneous (e.g., strata formation) and anisotropic (e.g., the elastic modulus varying with depth), but also there is the difficulty of evaluating the in situ stress-strain properties. Reliability-based method is on the horizon.
Necessity of a numerical analysis tool

- For practical application in design practice, a reliable standardized procedure has to be a combination of these two methods.
- The theory of elasticity: the basis for establishing approximate methods for predicting settlements for practical design where a computationally-efficient numerical procedure to estimate “representative” soil properties based on in situ tests answers to the all important question of selecting a soil stiffness (modulus) for use in these approximated results.
- Goal: simulate the soil-structure interaction in the field conditions where a proposed model would allow the engineer to easily calibrate and modify the modeling parameters by capturing the soil nonlinearity within a reasonable margin of conservative errors.
Winkler model

Overview of model components

Schematic diagram of primary model components

Euler beam theory

\[ \theta = \frac{dv}{dx} \quad (\tan \theta \approx \theta) \] (1)

\[ \kappa = \frac{d\theta}{dx} = \frac{d^2v}{dx^2} \] (2)

\[ EI \kappa = M \] (3)

\[ \frac{dM}{dx} = V, \quad \frac{dV}{dx} = -q, \quad \frac{d}{dx} \left( \frac{dM}{dx} \right) = -q \] (4a, 4b, 4c)

\[ \frac{d}{dx^2} \left( EI \kappa \right) = -q, \quad EI \frac{d^4v}{dx^4} = -q \] (5a, 5b)

\[ EI \frac{d^4v}{dx^4} = -q \] (6)

\[ q = -k_3v \] (7)

\[ EI \frac{d^4v}{dx^4} = k_4v \] (8)

Application of Euler Beam Theory
Linear Finite Element Analysis of Winkler problem

Discrete element approach

\[ E : \text{Modulus of elasticity, e.g., ACI eqn. } E = 57000 \sqrt{f_c} \]

\[ I : \text{Moment of inertia} = \frac{1}{12} b \cdot t^3 \]

\[ EI : \text{Flexural rigidity of Euler beam} \]

\[ k_s = k b l \]

\[ k : \text{the coeff. of subgrade reaction (kips/} in^3 \text{ or KN/} m^3 \text{)} \]

\[ l : \text{beam element length (ft or m)} \]

\[ b : \text{beam width (ft or m)} \]

Concept of discretization
Coefficient of subgrade reaction

- If a flexible foundation is to be analyzed, then it is recommended that Subgrade Modulus, i.e., the coefficient of subgrade reaction, be selected in consideration of geometry (B or L) and embedment depth (Terzaghi 1955 and Vesic 1961, respectively). This is because the value of subgrade modulus is not constant for a given soil but depends on length, width, and embedment depth of the foundation under consideration.

\[
\text{For square foundation on sand } k = k_{\text{wet}} \left( \frac{B+1}{2B} \right)^2
\]

\[
\text{For square foundation on clay, } k(\text{lb / in}^3) = k_{\text{wet}}(\text{lb / in}^3) \left( \frac{1(\text{ft})}{2B(\text{ft})} \right)^2
\]

where \( k_{\text{wet}} \) is result obtained from load tests by means of square plates measuring 1 ft X 1 ft.

\[
\text{For rectangular foundation of } B \times L, \quad k = k_{\text{square}} \left(1 + 0.5 \frac{B}{L}\right) \frac{2}{1.5}
\]
A combined footing that supports a three-column façade is subjected to service loads. Dimensions of the column is 1 m x 1m. The bearing soil is a 10-m thick medium-dense sand. Compute an immediate settlement due to service loads only.
FB-MultiPier Nonlinear Finite Element Analysis

Shallow foundation system is modeled using finite shell elements and soil springs.

A schematic sketch of FEA model of spreadfooting

Spring stiffness
FB-MultiPier Nonlinear FEA vs. linear (Winkler) FEA

Soil nonlinear behavior significantly affects the load-displacement behavior of the strip footing. Qu is about 38150 kN for a friction angle of 30 deg.

Variation of elastic settlement prediction
What if a factor of safety of 3 is used in design, i.e., a maximum service load is limited to $Qu/3=12716$ kN. That’s about a total applied load. But settlement is still too excessive (linear analysis predicts about 12 in settlement whereas nonlinear FEA predicts 18 in. settlement at 1/3 of $Qu$) and not acceptable.
FB-MultiPier Nonlinear FEA vs. linear FEA

Even with a factor of safety of 3, the predicted settlement is too excessive. If the settlement is limited to 2 in., an allowable load is 2086 kN (459 kips). Recall a total applied load of 13500 kN (3000 kips).
Interpretation of the numerical results

Limitation of linear elastic analysis

- Stiffness of soil is constant and independent of applied loads
  - The footing behaves almost like a rigid body (relatively speaking)
  - Deflection of the footing is predicted to be 0.33 m (13 in.)
  - The current footing design (size) is inadequate for serviceability
  - Considering an excessive deformation, reliability of the results is in question
  - Soil could be in near failure stages; what would be the ultimate bearing capacity of the footing?
  - Membrane stiffness of footing may be a contributing factor to flexure and, thus, deflection.

- FB-MultiPier solution captures nonlinear soil-structure interaction phenomena.
Parametric study (weak soil)

Loose sand with $k=1000$ kN/m$^3$ and Internal friction angle = 30 deg.

Deformation of a strip footing founded on soft soil
Parametric study (stiff soil)

Dense sand with $k=12000 \text{ kN/m}^3$ and Internal friction angle = 38 deg.

Deformation of a strip footing founded on stiff soil
Parametric study (stiff soil)
FB-MultiPier nonlinear FEA results
2. FB-MultiPier FEA model development

**FB-MultiPier** is a 3-D nonlinear FEA software program for use in bridge pier application. With a proven record of the validity, FB-MultiPier is widely used in analysis of bridge subfoundations both in U.S. and world-wide. A step-by-step procedure of FB-MultiPier shallow foundation model development is provided in the following.

Spreadfooting  
(from FHWA NHI-06-089, Dec./2006)
Example of FB-MultiPier FEA model development

- A single, rigid square foundation (118 in X 118 in) is to be constructed to support a column load. Assume that the supporting soil is a medium dense sand whose angle of internal friction, total unit weight, and subgrade modulus are equal to 32 degrees, 109 pcf, and 150 pci, respectively.
Step 1 : Select a problem type
Step 2: Select global parameters

- Go to "Analysis" page and specify a maximum number of iteration and the tolerance for a degree of accuracy of numerical solution. User must be familiar with the numerical solution procedure of FB-MultiPier in nonlinear analysis and choose an appropriate tolerance for problem of interest.
Step 3: Discretization of footing

- Change a pile-cap grid spacing in “Pile Edit” and locate a pile at the center of the pile cap. NOTE: At least one pile must be assigned. In Step 8, it will be explained how to make the axial resistance of this pseudo pile negligible and thus, its contribution to the bearing capacity of the foundation can be minimal.
Step 3: Discretization of footing

- Change a pile-cap grid spacing in “Pile Edit” and locate a pile at the center of the pile cap. NOTE: At least one pile must be assigned. In Step 8, it will be explained how to make the axial resistance of this pseudo pile negligible and thus, its contribution to the bearing capacity of the foundation can be minimal.
Step 4: Specify a footing elevation

- Change the cap (footing) elevation accordingly to the ground surface. NOTE: The pile cap is modeled using finite thin shell elements. The centerline elevation of the cap (the shell elements) must be properly located to include the half of the cap thickness. The length of a pseudo pile have to be at least equal to the pile cap thickness: by default, the program expects to have at least one pile embedded in the soil.
Step 5: Choose “Bearing Resistance” option

Step 6: Specify material properties of footing
“Bearing Resistance” option
Step 7: Specify soil material properties

Select a lateral py model of medium dense sand of interest. The bearing stiffness in the soil-cap interaction is controlled by the lateral py model of the FB-MultiPier program; internal friction angle and subgrade modulus are the key parameters in order to compute the bearing stiffness of cohesionless soil whereas undrained shear strength (= cohesion) is used as to compute the bearing stiffness for cohesive soil.
Step 8: Modify the axial soil models

A pile length of 5 ft is chosen in the model. Set values for ultimate skin friction of the soil axial model (tz curve) and axial bearing failure load of the soil tip model (qz curve) equal to a small magnitude, e.g., 0.001 psf and 0.001 kips. By doing so, the axial resistance of the pile becomes negligible. At minimum, one pile must be included in the model.

Axial properties (T-z curve)

Tip properties (Q-z curve)
Step 9: Validation of the results

For this particular example, FB-MultiPier solution process can converge with a tolerance of 0.01 kip of which the foundation is subjected to an axial load of 200 kips and resulting displacement is 0.1076 in. With an applied load greater than 1000 kips, convergence fails for a tolerance of 0.1 kip as the theoretical ultimate bearing capacity of the foundation is computed as 0.0720 ksi, which is equivalent to an applied load of 1002.5 kips to a square shallow foundation with a size of 118 in X 118 in.

SUM OF TOTAL SOIL SPRING LOADS

<table>
<thead>
<tr>
<th>CHECK: Total Load Carried by the Soil</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Sum of NF+FF Soil Spring Loads)</td>
</tr>
<tr>
<td>X Direction = 0.0000 Kips</td>
</tr>
<tr>
<td>Y Direction = 0.0000 Kips</td>
</tr>
<tr>
<td>Z Direction = 200.0229 Kips</td>
</tr>
<tr>
<td>Pile Cap Bearing = 200.0128 Kips</td>
</tr>
<tr>
<td>Sum of Tip Forces = 0.0001 Kips</td>
</tr>
</tbody>
</table>

Avg. Ult. Bearing Capacity (Soil-Cap Interaction) = 0.0720 Kips/in^2
Computed using Meyerhof Method (1962)

Avg. Ult. Uplifting Capacity (Soil-Cap Interaction) = 0.0000 Kips/in^2
Based on Meyerhof and Adams (1968), Das (1980)
Step 9 : Validation of the results

According to the Meyerhof’s general bearing capacity equation, an analytical solution is calculated as follows. Recall that:

\[
q_u = \frac{A}{cN_cF_{eq}F_{cd}F_{et}} + \frac{B}{qN_qF_{eq}F_{cd}F_{et}} + \frac{C}{BN_yF_{y}F_{yd}F_{yi}}
\]

Term A = 0 since \( c = 0 \).

Term B = 0 since \( q = 0 \).

For Term C, Meyerhof’s bearing capacity factor would be equal to

\[
N_y = (N' - 1) \tan(1.4\phi) = (23.1770) \tan(1.4\cdot32) = 22.0227
\]

However, it is necessary for a conservative design to incorporate allowance of local shear failure of soil in a loose to medium density state in the prediction of the ultimate bearing capacity. Thus, using the theory presented by Peck et al. (1953), the bearing capacity factor is modified and subsequently used in the FB-MultiPier analysis, which is \( N_y = 14.5869 \).

Therefore,

\[
\text{Term } C = 0.5 \cdot (109 \cdot 257) \cdot (118 \text{ tn}) \cdot (14.5869) \cdot (1.3255) \cdot (1.0) \cdot (1.0) = 0.0720 \text{ kN}
\]

where

\[
F_{y} = 1 + 0.1 \left( \frac{B}{L} \right) \tan^2(45 + \frac{\phi}{2}) = 1 + 0.1(45.0)(3.2546) = 1.3255
\]

\[
F_{yd} = 1.0
\]

\[
F_{yi} = 1.0
\]
Step 10: Interpretation of the data

The data from all shell elements consist of shear and moment. It is important to note that the moments and shear results are per unit length of plate. For example, unit of moment is kN·m per m (or kip·in per in in US customary unit) and unit of shear is kN per m (or kips per in).

![FB-MultiPier 3-D results window](image)

![Moment (My) contour](image)
3. Theoretical basis of FB-MultiPier FEA model

Theory and Implementation of nonlinear soil-structure interaction

3.1 Computation of soil resistance: Newmark’s solution of Boussinesq’s equation
3.2 Constitutive relationship: Hyperbolic model by Duncan and Chang
3.3 Homogeneization of heterogeneity: Averaging methods

Three-dimensional view of variation of stress in elastic medium
3.1 Computation of soil resistance (1/3)

Newmark’s load characterization: stress under a rectangular area of uniform contact pressure by Boussinesq (1885)

\[
\Delta \sigma_z = \int_{y=0}^{L} \int_{x=0}^{B} \frac{3q(dx\,dy)z^3}{2\pi(x^2 + y^2 + z^2)^{5/2}}
\]

\[
\Delta \sigma_z = q I
\]

\[
I = \frac{1}{4\pi} \left( \frac{2mn\sqrt{m^2 + n^2 + 1} \cdot m^2 + n^2 + 2}{m^2 + n^2 + m^2n^2 + 1} + \arctan \frac{2mn\sqrt{m^2 + n^2 + 1}}{m^2 + n^2 + 1 - m^2n^2} \right)
\]

if \( m^2 + n^2 + 1 \geq m^2n^2 \)

\[
I = \frac{1}{4\pi} \left( \frac{2mn\sqrt{m^2 + n^2 + 1} \cdot m^2 + n^2 + 2}{m^2 + n^2 + m^2n^2 + 1} + \pi + \arctan \left( \frac{2mn\sqrt{m^2 + n^2 + 1}}{m^2 + n^2 + 1 - m^2n^2} \right) \right)
\]

otherwise

\[
m = \frac{B}{z} \quad n = \frac{L}{z}
\]
3.1 Computation of soil resistance (2/3)

Stress superposition method is applied to analytical solution of Boussinesq’s equation by Newman (1935)

Superposition technique

Variation of the influence factors beneath the corner (red) and the center (blue)
3.1 Computation of soil resistance (3/3)

Variation of induced vertical compressive stresses beneath a rectangular shallow foundation

Three-dimensional view of variation of stress in elastic medium
3.2 Constitutive relationship: hyperbolic model

\[ \sigma_1 - \sigma_3 = \frac{\varepsilon}{A \varepsilon + B} \]

Kondner’s original model (1963)

\[ \Delta \sigma_z = \frac{\varepsilon}{\frac{1}{E_i} + \frac{\varepsilon}{P_u}} \]

Duncan and Chang model (1970)

Hyperbolic stress-strain relationship
3.2 Constitutive relationship

Duncan and Chang (1970) model is conceptually simple and computationally robust

- The two parameters of this nonlinear stress-strain relationship can be directly obtained from drained triaxial compression test of both cohesive and cohesionless soil whereas the model parameters are also abundantly available in the literature

- Limitation
  - Numerical instability may occur when stress approaches shear failure
  - No volume change due to shear stress is considered, i.e. shear dilatancy
  - Input parameters must be selected appropriately for soil conditions; what if Non-homogeneous soil condition exists
  - Quasi-static analysis only
3.3 Homogenization of heterogeneity

Averaging techniques of the elastic modulus of the soil is evaluated over a depth of the shallow foundation

- Bowles’ weighted averaging method (shown in the right)
- Equivalent thickness method (predicts very comparable results to Texas A&M load test)
- Anisotropy averaging method (great for strain influence method; compatible for strain energy approach)

\[ E_h = \frac{1}{H} \int_0^H E(z) \, dz = \frac{1}{H} \sum_{i=1}^{N} (E_i h_i) \]
On-going efforts at BSI

• Goal
  • Develop a versatile, easy-to-use computational tool for design engineers

• Objectives
  • Schemertmann’s influence line method is being implemented in FB-MultiPier – expected release date: May/2012
  • Make both stress and strain influence methods available for engineers to minimize uncertainty in characterization of soil stiffness
  • Provide engineers with a well-documented validation using relevant in-situ and load test results, i.e., a degree of the validity of the numerical model
  • Refine the tool based on feedback from the engineers – continual efforts.